Name: Final Exam – MA 242-002 Summer 1 2019

You may earn up to 210 points on this exam, but it will be scored out of 200. You have 3 hours to complete this exam. You may not use a graphing calculator or any communication device during the test. You may use a scientific non-graphing calculator. Good luck!

1. (15 pts) Consider the line connecting the origin to $P(1, 2, 3)$ and the line connecting $P(1, 2, 3)$ to $Q(1, 0, 1)$. Find the equation of the plane containing both of the lines.

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2. (18 pts) Consider the helical trajectory given by the vector-valued function $\vec{r}(t) = \langle 10t, 7\cos t, 7\sin t \rangle$. Compute the unit tangent vector $\hat{T}(t)$ and the unit normal vector $\hat{N}(t)$.

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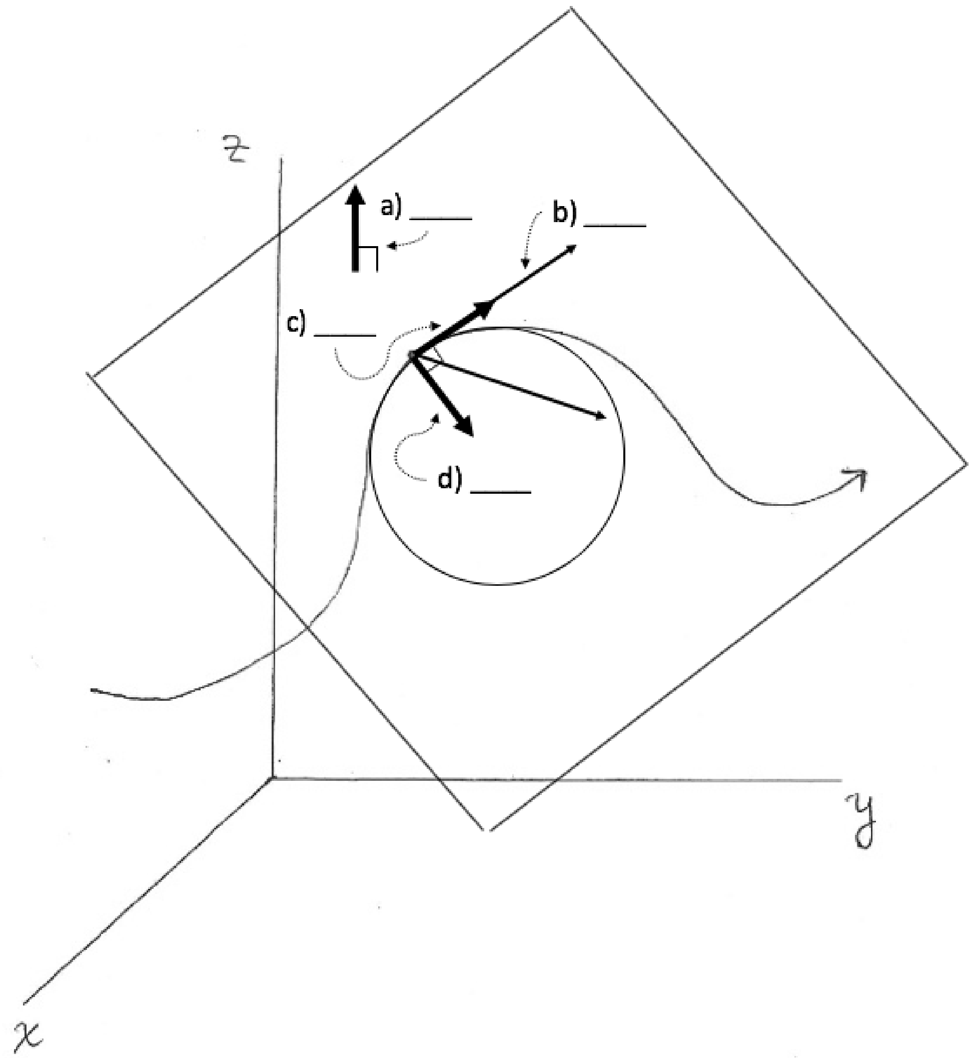
3. (8 pts) Label the following diagram with the names of the vectors indicated.



4. (7 pts) If at the point $\vec{r}(t_0) = \langle 1, 2, 3\rangle$, the center of the osculating circle is $P(4, 5, 6)$, find the curvature $\kappa(t_0)$.

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5. (18 pts) Consider the function $f(x, y) = x^2 - 2x + y^2 - 4y + 7$.

a) Perform two iterations of gradient descent on $f(x, y)$ with a learning rate $\delta = 1/4$ starting from the point $(x_0, y_0) = (2, 4)$.

b) Approximate the value of $f(2.1, 3.9)$ by linearizing f about the point $(2, 4)$.

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6. (20 pts) Use the method of Lagrange Multipliers to find the maximum of the product of two numbers x and y given that (x, y) is a coordinate pair in the first quadrant located on the ellipse given by:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

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7. (10 pts) Evaluate the triple integral with as little work as possible.

$$\int_0^3 \int_0^2 \int_0^1 z e^{x+y+z^2} \, dz \, dy \, dx$$

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8. (18 pts) Evaluate the triple integral by changing to a more convenient coordinate system. Sketch the region of integration Ω and describe it in the target coordinate system, then set up and evaluate a new integral.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{5} z \, dz \, dy \, dx$$

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9. (20 pts) Let $\mathcal{S}$ be the surface of the sphere of radius R centered at the origin. Give a parametrization of the sphere, then evaluate the surface integral

$$\iint_{\mathcal{S}} dS$$

to confirm that its surface area is $A(\mathcal{S}) = 4\pi R^2$.

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In problems #10-13, if you use any theorems you must state the theorem and identify it by name before you use it in order to receive full credit.

10. (20 pts) For each of the following, determine if $\vec{F}$ is conservative. Then evaluate $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$.

   (a) $\vec{F} = \langle yz, xz, xy \rangle$ and $\mathcal{C}$ given by $\vec{r}(t) = \left\langle 2t^2, e^{1-t^2}, \arctan(\frac{t^2}{2}) \right\rangle$ for $0 \leq t \leq \sqrt{2}$

   (b) $\vec{F} = \left\langle \sqrt{\frac{y}{x}}, \sqrt{\frac{x}{y}} \right\rangle$ and $\mathcal{C}$ given by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $0 \leq t \leq 2\pi$

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11. (20 pts) Let $\mathcal{S}$ be the surface given by the portion of the inverted cone $z = 5 - \sqrt{x^2 + y^2}$ that lies above the $z = 0$ plane (*Note:* $\mathcal{S}$ does not include the circular base, just the rounded face of the cone.). Evaluate:

$$\iint_{\mathcal{S}} \nabla \times \vec{F} \cdot d\vec{S}$$

where $\vec{F}$ is given by the following:

$$\vec{F} = \left\langle z^2 - y, z^3 + x, \ln(xy) \arctan(z) \right\rangle$$

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12. (17 pts) Let C be the closed, counterclockwise-oriented curve given by the Archimedean spiral $r = \theta$ for $0 \le \theta \le 2\pi$ and portion of the x-axis lying between $x = 0$ and $x = 2\pi$ (see figure). Let $\vec{F} = \left\langle -\frac{1}{2}y + e^x, \frac{1}{2}x + e^y \right\rangle$. Evaluate:

$$\oint_C \vec{F} \cdot d\vec{r}$$

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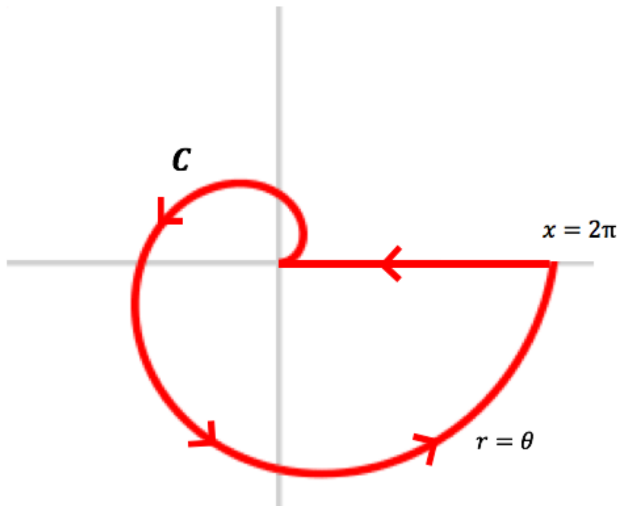

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13. (15 pts) Let $\vec{F} = \langle y^3 \ln z, \sqrt{z}e^x, 4\arctan(xy) + \pi z \rangle$. Let $\mathcal{S}$ be the unit sphere centered at the origin. Compute the flux:

$$\oiint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$$